Tip Sheet

Remediation for Mathematical Difficulties

Individuals with learning difficulties related to maths may frequently make errors when reading, writing and recalling numbers; experience difficulty with abstract concepts (e.g. time and direction); struggle to remember maths facts, rules and formulas and sequences; have difficulty applying operational procedures accurately and consistently; and show limited strategic planning ability. These individuals will require explicit, understanding-based and carefully structured instruction on basic number concepts in order to develop mathematical fluency (proficiency).

Mathematical proficiency is the compilation of three foundational skills:

- **Accuracy** – the individual records work correctly, knows important number relationships, retrieves number facts accurately, and the method chosen yields the correct solution.
- **Flexibility** – the individual is able to approach the problem through a variety of different means, that is, they are able to be flexible in order to choose an appropriate strategy for the question and numbers involved, and they are also able to use a different method to check their results if necessary.
- **Efficiency** – the individual can carry out steps easily and in a reasonable amount of time, and they are able to keep track of sub-problems. For example, some individuals who get the correct solution can take significantly longer than their peers to solve the problem, they may use a convoluted method to facilitate the calculation process, or they do not feel confident with their answer and often recheck their working.

Proficiency in mathematics is only achieved when all three components are present, and for most individuals, this is only achieved as a result of high-quality numeracy instruction.

Guiding Principles for Effective Maths Instruction

Individuals may require different degrees and types of intervention at various stages of their mathematical development. There are a number of key factors to consider when choosing or putting into place an effective intervention program for maths, including:

- Instructional explicitness.
- The use of clear and precise explanations of logically sequenced instructions.
- Inclusion of a strong conceptual basis for any procedures/processes that are taught.
- Cumulative review (i.e. review the new skill, as well as those on which it is based or related).
- Support for motivation, attention or self-regulation.
- Ongoing progress monitoring.
- Specific to the needs of the student.

In addition to the guiding principles, high quality maths instruction should ensure that individuals are taught foundational mathematics knowledge; teaching occurs in small, progressive steps; careful consideration is paid to cognitive load and memory demands; an intensive, cyclical teaching program is used; and individuals are guided from concrete to abstract.

The following pages outline the foundational mathematics knowledge that all students require in order to develop mathematical proficiency, as well as strategies to teach these skills and understandings.

Teaching Foundation Knowledge

Often the focus of numeracy remediation is on the development of broader mathematics skills, such as algebra and geometry. However, many individuals actually experience maths difficulties as a result of weaknesses in the underlying numeracy skills that are required to effectively complete more applied mathematical tasks. Therefore, remediation should focus on ensuring that the following early numeracy skills which are paramount to successful maths development are well-consolidated and automatic.

Tip Sheet

Remediation for Mathematical Difficulties (continued)

1. Number Sense

Number sense is best understood as our innate ability to make sense of numbers. It involves understanding how our number system works and having an intuitive feel for the size and magnitude of numbers. Strategies to support the development of number sense include:

- **Subitising activities** – being able to identify and name the number of objects (up to four) in a small group without counting:
  - Scatter a small number of items (e.g. counters, blocks) and have the individual name the number of items within the group without counting.
  - Complete the same activity using dot patterns, gradually increasing the size of the set.
  - Formally teach dot patterns using counters of the same colour.
  - Play games involving dice and have the individual name the number on the die without counting.
- **Estimating activities** – being able to roughly judge how many objects are in a group:
  - Teach the concepts of "ish" as the basis of estimation.
  - Play estimating games with counters or objects (start with five to ten, and gradually increase the number)
  - Play estimating games whilst completing everyday activities (e.g. estimate how many steps it will take to get from one location to another, or how many times they can complete an action in a set time limit).
  - Use objects of a set size (e.g. blocks) to estimate the length or height of another object.
  - Use number lines to estimate where a number will fall.

2. Counting Strategies

In order to count effectively, individuals must have knowledge of the counting principles – that is, we count each object only once, we say counting words in a set order, the last counting word we say represents the total number or value of the set, we can count any collection of objects, and objects can be counted in any order. Strategies to support the development of counting include:

- Practicing oral counting, as well as counting real objects.
- Counting forwards by ones, and then transition to counting backwards.
- Encouraging students to count using number tracks, and then introduce number lines.
- Practicing counting various objects in a variety of orders and directions.
- Using concrete materials when counting.
- Practicing reading and writing numbers.
- Carefully build up knowledge of the way our number system is constructed and target areas of difficulty (e.g. when numbers change from one decade to another).

3. Procedural Knowledge

Individuals require a strong foundation in procedural knowledge, that is, how to go about completing a calculation, in order to easily apply series of steps to solve a given mathematical problem. Additionally, well developed procedural knowledge allows individuals to approach more difficult sums and tasks in a strategic and flexible manner. Strategies to support the development of procedural knowledge include:

- Using concrete materials to build numbers and explore the physical size of quantities.
- Teaching dot patterns to assist with early calculations and number relationships.
- Teaching number bonds up to 10.
- Using concrete materials to teach place value, including the use of a place value grid.

Tip Sheet

Remediation for Mathematical Difficulties (continued)

- Practicing step counting.
- Encouraging individuals to estimate first and then check their answer.
- Using graph paper to assist with setting out written sums.
- Encouraging individuals to use talk-aloud strategies to discuss the procedure they have used.

4. Number Facts

Number fact knowledge is essential for mathematical fluency. In order for an individual to develop automatic recall of number facts, they must have a conceptual understanding of the procedures at hand (e.g. they need to understand the process of addition or subtraction), have the ability to solve the sum accurately and consistently in an untimed condition, and understand the commutative property of sums (e.g. adding 3 and 4 will give the same answer as adding 4 and 3). Strategies to support the development of number fact knowledge include:

- Teaching a limited number of facts at a time, adding new facts once the previous facts have been mastered.
- Making practice cumulative – introducing new facts along with those previously mastered.
- Keeping practice sessions short.
- Reviewing the commutative property of number facts to reinforce relationships.
- Helping children see connections between number facts (e.g. if 5+5 = 10, then 5+6 = 11).
- Reducing the emphasis on speed in the early stages.

5. Language of Maths

There are many words used in maths that have completely different meanings when used in other contexts. Individuals need to learn that some words or phrases have a specific meaning when used in the context of maths. Difficulties understanding the language of maths can result in poor comprehension of worded questions, confusion about the procedure that is needed to solve a maths problem, or failing to identify key information. Strategies to support the development of maths vocabulary include:

- Encouraging individuals to use concrete materials to clarify worded questions.
- Asking individuals to rewrite word problems in simpler language.
- Developing a word wall or maths dictionary and encouraging students to record new words.
- Using consistent language in the early stages of teaching and gradually introducing new words that have the same meaning.
- Using or creating stories to introduce new maths vocabulary
- Using graphic organisers or Frayer boards to explore the language of maths.
- Using word sorts to link words and concepts.
- Encouraging students to provide verbal and written explanations of mathematical concepts.

Teaching Problem solving Skills

Teaching problem solving strategies is an essential evidence-based strategy for supporting students with mathematical learning difficulties and disorders. To become a mathematical problem-solver, students must apply their knowledge of mathematical concepts and foundational skills and understand the relationships between the two. Students also need to recognise and create problems from real-world situations and apply appropriate problem-solving strategies to determine a solution. In many cases, word problems are used to assess a students' problem-solving abilities. Research has consistently shown that effective instruction in word problem solving adheres to the following guidelines:

Tip Sheet

Remediation for Mathematical Difficulties (continued)

1. Avoid tying key words to mathematical operations

Teaching students to identify key or target words for solving problems has been shown to be less effective. When students are tying key words to operators, they often seek out these key words and apply the mathematical procedure in a rote learning manner without thinking conceptually about what the problem is asking or understanding why they are performing that procedure.

2. Ensure that students have learnt an attack strategy to help guide the process of problem solving

Before approaching a word problem, students with learning difficulties may need additional support to ensure that they understand the problem being asked, can monitor their application of mathematical concepts and procedures, and can check their answers. An attack strategy provides a tool for students to use to assist with structuring their thinking before, during and after solving a problem. Students with mathematical learning difficulties need explicit instruction to learn how to apply attack strategies to worded problems. Attack strategies often consist of metacognitive strategies such as mnemonics and/or acronyms to prompt student recall. There are a variety of mnemonics and acronyms that can be used, however, most share four important components:

- **Understand** – carefully reading the problems and determining what the problem is asking.
- **Plan** – consider the problem type and what information is required and plan out the necessary procedure to find the solution.
- **Carry Out** – Complete the calculation
- **Review** – Check the answer and reflect on whether the answer makes sense.

3. Provide direct instruction in recognising and solving word problem types

To ensure success in planning and carrying out mathematical procedures, students with learning difficulties and disorders will also benefit from explicit instruction in identifying the type of worded problem based on the structure of the problem (schema). This then informs the procedure used to solve the problem and produce an answer. In general, most worded problems fall into one of six categories (or schemas) including: total, difference, change, equal groups, compare, and ratios/proportions.

Providing Multiple Representations

When considering the diverse needs of students with mathematical learning difficulties and disorders, the use of the concrete-representational-abstract (CRA) sequence has consistently been shown in research to be an effective and necessary process in all mathematical teaching. The use of the CRA sequence helps deepen students' understanding of mathematical concepts and provides an opportunity for students to visualise and experience the mathematical concept or procedure. The CRA approach involves carefully guiding students from the concrete to abstract form of mathematics. Students in the early stages of learning are provided with concrete materials to feel and build the maths concept. As they demonstrate a good understanding of the new concept, students are then encouraged to pair the use of concrete materials with semi-concrete and abstract, gradually fading out the use of concrete materials as they become more proficient.

- **Concrete**: includes the use of three-dimensional, hands-on materials and manipulatives that students can touch and move to promote understanding of different concepts and procedures.
- **Representational (semi-concrete)**: includes the use of two-dimensional pictures, images, or virtual manipulatives. Often these materials include pictures of concrete materials and can also include graphic organisers and charts.
- **Abstract**: consists of numbers, symbols, and words, and reflects the typical view of mathematics.